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Basic Statistical Tolerancing

In building an assembly from parts, the Quality Engineer needs to understand how the tolerances of the individual parts impact the tolerance of the final assembly. The two common methods are worst case analysis and statistical tolerancing. This paper compares the methods.

Statistical tolerancing is a large field with many diverse aspects. This paper discusses statistical tolerancing using some simplifying assumptions explained below. More advanced statistical issues, such as using non-normal distributions are beyond the present scope.

A Simple Illustration

One assembly in our production process stacks ten metal disks, one on top of another. Each disk is 1 inch in diameter and 0.125 inches thick. We need a stack height of 1.250 ± 0.010 inches. What tolerance should we assign to each disk?

Notice this specification includes a target value of 1.250" and a bilateral symmetric tolerance of 0.010". The specification is bilateral because there is a specification on each side, upper and lower. It is symmetric because it is the same on both sides.

This specification method defines an interval from 1.240" to 1.260", denoted [1.240", 1.260"]. Any stack inside the interval is conforming, while any stack outside the interval is nonconforming.
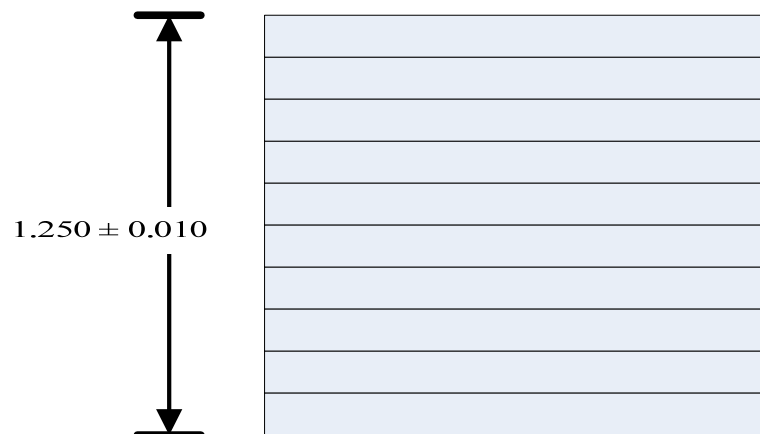

Figure 1 The Stack of 10 Disks

The Worst Case Method
One method, often called the Worst Case method, assumes first, that every part is at the low end of the specification and then every part is at the high end. In this case, with ten disks, we can easily divide the stack tolerance, 0.010", by 10 to get the tolerance for each disk. In this case, the disk specification is 0.125 ± 0.001 inches. If all the disks are at the low end of the tolerance, 0.124", the stack height is $10 \times 0.124" = 1.240"$. Similarly, for all disks at the high end, 0.126", the stack height is $10 \times 0.126" = 1.260"$. Under these conditions, every stack will be conforming.

Notice the underlying theory in this method. To determine the lower specification limit of the stack, we added the lower specification limit of each component, the target of the stack is the sum of the component targets, and the upper specification is the sum of component upper specifications. We can express this is

$$Y = f(x_1, x_2, \ldots, x_n)$$

where each x_i represents a component.

In this case we add, for example, the lower specification limits (LSL) to obtain the stack's LSL.

$$Y_{LSL} = \sum_{i=1}^{n} x_{i_{LSL}} = \sum_{i=1}^{10} 0.124 = 1.240$$

While this method works, the results are often conservative and may create problems with process capability. The tight tolerances may exceed the capability of the process to hold them. One solution is statistical tolerancing.

Describing Processes
Statistical tolerancing recognizes that processes have variability and the probability of selecting disks that are all at the same extreme end of the distribution is rare. To see this, assume that the process produces 1.0% of the disks at the low end of specification. Selecting the disks at random, we can calculate the probability that all 10 disks are at the low end.

$$0.01 \times 0.01 \times \ldots \times 0.01 = 0.01^{10} = 0.00000000000000000001$$

We can use the probability distribution to our advantage. First we will make some assumptions about the process to manufacture the disks. Figure 2 shows these assumptions.
- The process is centered at the target, *i.e.,* 0.125".
- The process output, the disks, is normally distributed.
- The process output distribution fits exactly inside the specification.
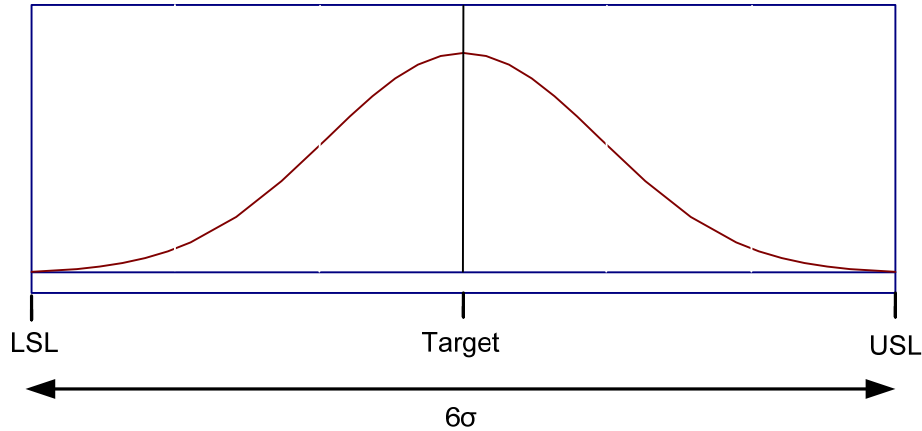
Figure 2 A Normal Distribution That Exactly Fits the Specification

In this case, with a centered distribution that exactly fits the specifications, we know the C_p and C_{pk}! They are both 1.00.

We can use our process distribution knowledge to approach the problem in a different way. Instead of adding component target or specification limits, we can "add" the distributions. The same basic model applies, but we combine the inputs in a different way.

$$Y = f(x_1, x_2, \ldots, x_n)$$

Here are the basic rules for combining normal distributions.

> If two parts are additive, then add the means and the variances (not the standard deviation) of the normal distributions.

> If two parts are subtractive, then subtract the means and add the variances (not the standard deviation) of the normal distributions.

Remember that the variance is the square of the standard deviation.

Example 1: An Additive Case
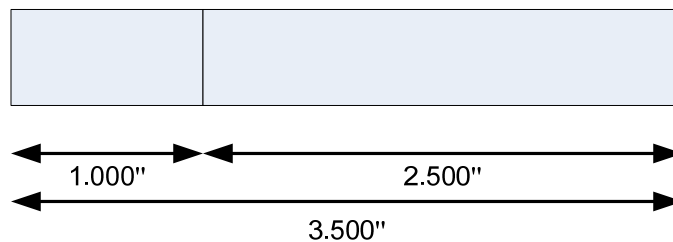We combine the lengths of two parts to make an assembly, as shown in the figure.

1.000"   2.500"

3.500"

Figure 3 An Additive Example

Assume Part A's normal distribution has a mean of 1.000" and a standard deviation of 0.03" (μ_A = 1.000", σ_A = 0.03"). For Part B, we have μ_B = 2.500", σ_B = 0.02".

We expect the distribution of the final assembly to be

$$\mu_F = \mu_A + \mu_B = 1.000 + 2.500 = 3.500$$

$$\sigma_F = \sqrt{\sigma_A^2 + \sigma_B^2} = \sqrt{0.03^2 + 0.02^2} = 0.036$$

Example 2: A Subtractive Case
Our assembly uses the difference between two dimensions, as shown by the extension length of the pin from the hole as shown in the figure.

For these parts, assume we have the following:
       Pin μ_P = 0.60" σ_P = 0.03"
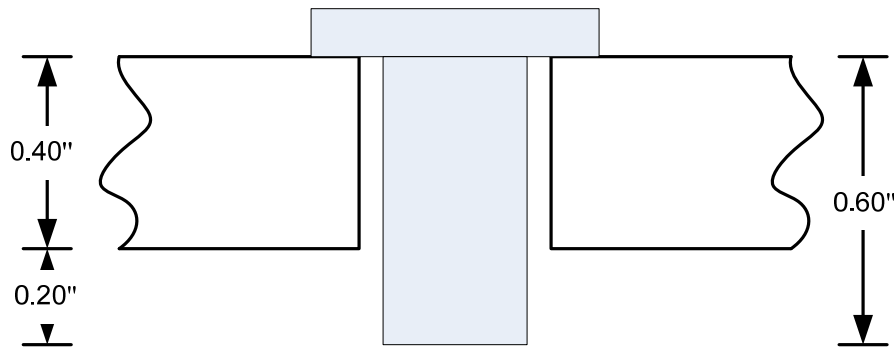   Hole μ_H = 0.40" σ_H = 0.02"

Figure 4 A Subtractive Example

We can expect the pin's extension distribution to be

$$\mu_E = \mu_P - \mu_H = 0.60 - 0.40 = 0.20$$

$$\sigma_E = \sqrt{\sigma_P^2 + \sigma_H^2} = \sqrt{0.03^2 + 0.02^2} = 0.036$$

Statistical Tolerancing
With this method of adding distributions, we can return to the stack of ten disks. Assume we desire a C_{pk} = 1.00 for the stack height of 1.250 ± 0.010 inches. With a centered process we know that 6σ fills the specification width. We can calculate the mean and standard deviation of the stack.

$$\mu_S = 1.250$$

$$\sigma_S = \frac{0.020}{6} \approx 0.00333$$

Using the additive model described above we can calculate the distribution for the disks. We make the same assumptions of a centered process and $C_{pk} = 1.00$.

The mean and standard deviation of the disk process should be:

$$\mu_D = \frac{1.250}{10} = 0.125$$

$$\sigma_D = \sqrt{\frac{(0.020/6)^2}{10}} = 0.00105$$

Under these conditions the specification for the disk should be 0.125 ± 0.003 inches.

Compare this with the worst case analysis were we concluded the specification should be 0.125 ± 0.001 inches. The statistical tolerance method yields a wider specification and a better opportunity for a capable process.

The Risk
There is a risk associated with statistical tolerancing compared to worst case. The final assembly may be out of specification. The normal distribution has a small, but finite, probability that a value will fall into one of the tails of the distribution. In this case, a nonconforming final assembly is either too big or too small, as compared to the specification. Since we set the specification at the ±3 standard deviation points, we can calculate the probability of being outside. These are the area in tails beyond the upper and lower three standard deviation points; it is 0.27% or 27 out of every 10,000.

Using selective assembly, *i.e.,* replacing a part with another one, the risk can drop considerably.