The OC Curve of Attribute Acceptance Plans

The Operating Characteristic (OC) curve describes the probability of accepting a lot as a function of the lot's quality. Figure 1 shows a typical OC Curve.
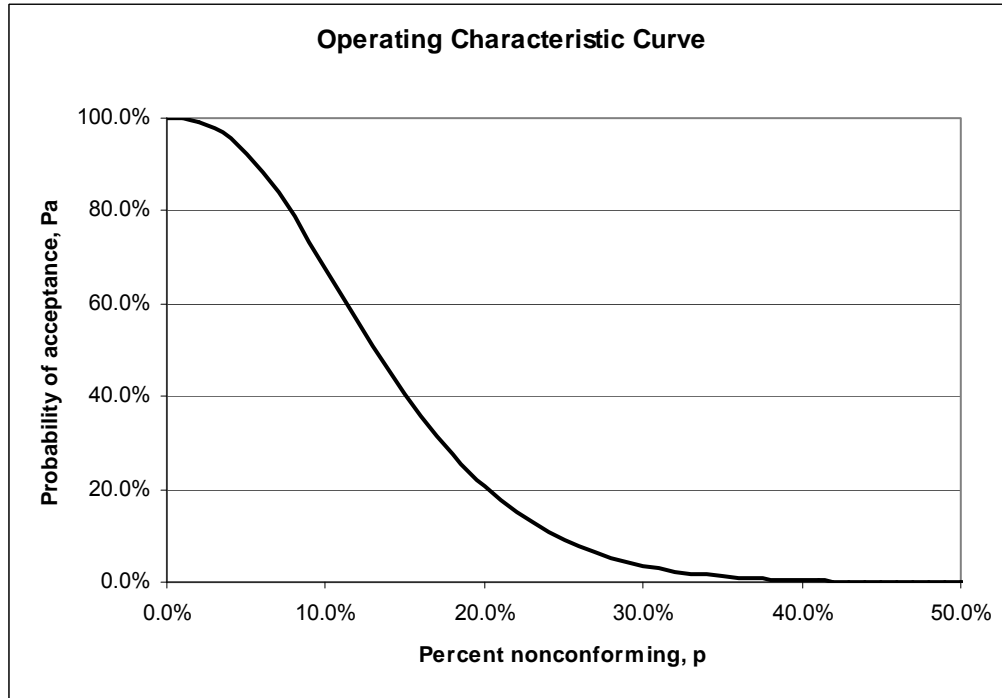

Figure 1 Typical Operating Characteristic (OC) Curve

The Shape of the OC Curve

The first thing to notice about the OC curve in Figure 1 is the shape; the curve is not a straight line. Notice the roughly "S" shape. As the lot percent nonconforming increases, the probability of acceptance decreases, just as you would expect.

Historically, acceptance sampling is part of the process between a part's producer and consumer. To help determine the quality of a process (or lot) the producer or consumer can take a sample instead of inspecting the full lot. Sampling reduces costs, because one needs to inspect or test fewer items than looking at the whole lot.

Sampling is based on the idea that the lots come from a process that has a certain nonconformance rate (but there is another view described below). The concept is that the consumer will accept all the producer's lots as long as the process percent nonconforming is below a prescribed level. This produces the, so called, ideal OC curve shown in Figure 2.

When the process percent nonconforming is below the prescribed level, 4.0% in this example, the probability of acceptance is 100%. For quality worse than this level, higher than 4%, the probability of acceptance immediately drops to 0%. The dividing line between 100% and 0% acceptance is called the Acceptable Quality Level (AQL).

The original idea for sampling takes a simple random sample, of *n* units, from the lot. If the number of nonconforming items is below a prescribed number, called the acceptance number and denoted *c*, we accept the lot. If the sample contains more nonconforming items, we reject the lot.
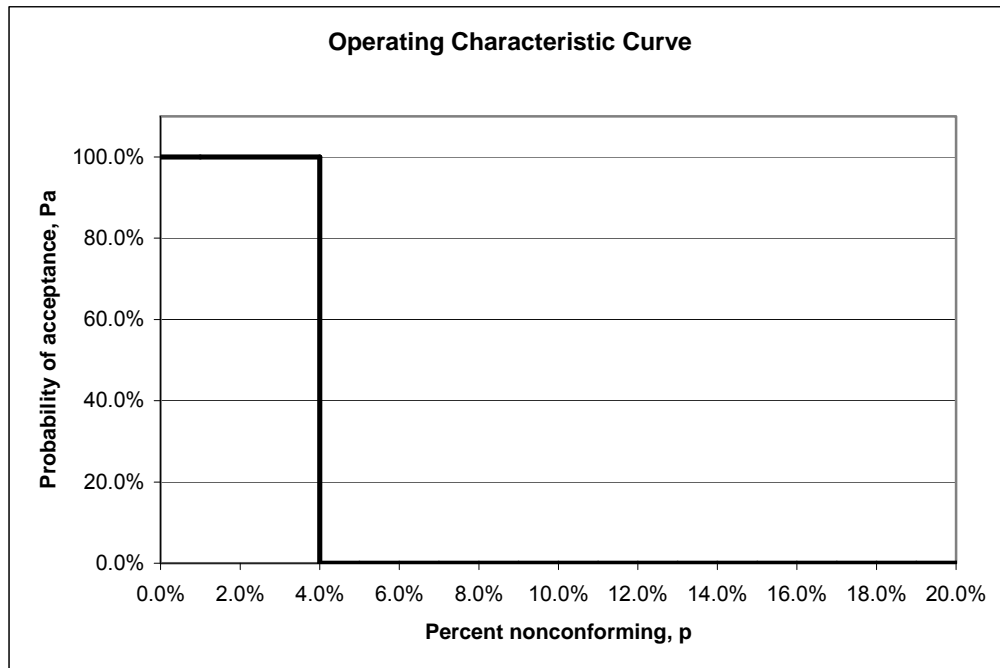

Figure 2 Ideal OC Curve

The only way to realize the ideal OC curve is 100% inspection. With sampling, we can come close. In general, as the sample size increases, keeping the acceptance number proportional, the OC curve approaches the ideal, as shown in Figure 3.

Similarly, as the acceptance number, *c*, gets larger for a given sample size, *n*, the OC curve approaches the ideal. Figure 4 illustrates the relationship.

Some Specific Points on the OC Curve
Because sampling doesn't allow the ideal OC curve, we need to consider certain risks. The first risk is that the consumer will reject a lot that satisfies the established conditions, *i.e.,* the process quality is acceptable, but, by the luck of the draw, there are too many nonconforming items in the sample. This is called the producer's risk, and is denoted by the Greek letter α.

The second risk is that the consumer will accept a lot that doesn't meet the conditions, *i.e.,* by the luck of the draw there are not many nonconforming items in the sample, so the lot is accepted. This is the consumer's risk and is denoted by the Greek letter β.

The literature contains a variety of typical values for α and β, but common values are 5% and 10%. When we locate these values on the OC curve, expressed in terms of probability of acceptance, we actually locate $1 - \alpha$.
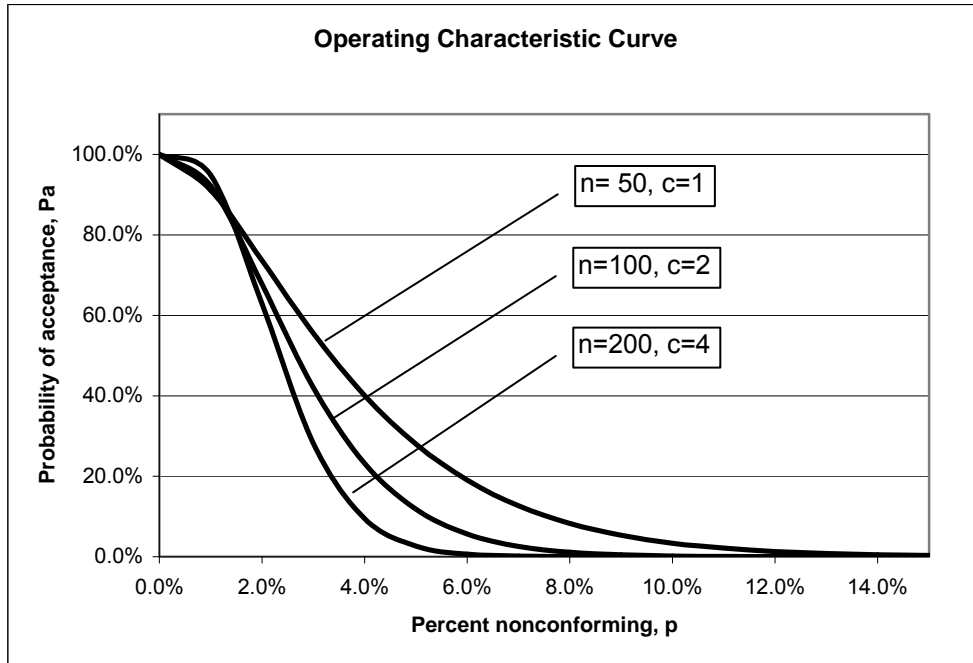
Operating Characteristic Curve

Probability of acceptance, Pa

n= 50, c=1

n=100, c=2

n=200, c=4

Percent nonconforming, p

Figure 3 As *n* Increases the OC Curve Approaches the Ideal

Operating Characteristic Curve

Probability of acceptance, Pa

n=100, c=2

n=100, c=1

n=100, c=0
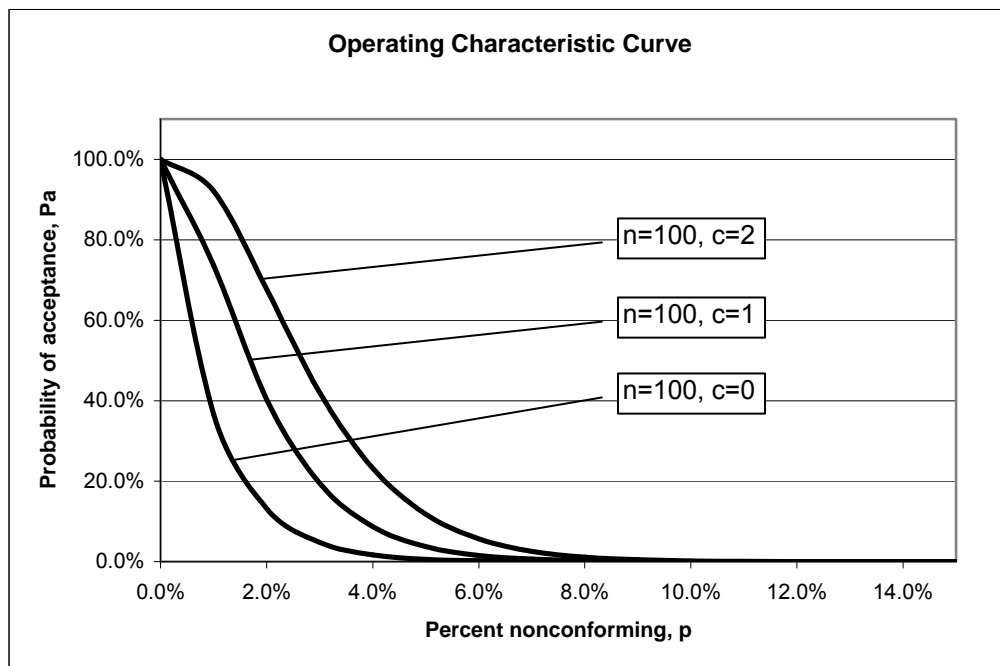
Percent nonconforming, p

Figure 4 As *c* Increases, for Fixed *n*, the OC Curve Approaches the Ideal

These points correspond to specific values of lot quality and they have a variety of names. The point associated with $1 - \alpha$ is often called the Acceptable Quality Limit or AQL. This is not necessarily the same AQL used to describe the ideal OC curve. For an α of 5% this means a process operating at the AQL will have 95% of its lots accepted by the sampling plan.

Similarly, the point associated with β is often called, in contrast, the Rejectable Quality Limit or RQL. A process operating at the RQL will have 5% of its lots accepted by the sampling plan.

Lastly, some authors consider the process quality where the lots have a 50% probability of acceptance. This is called the Indifference Quality Limit or IQL.

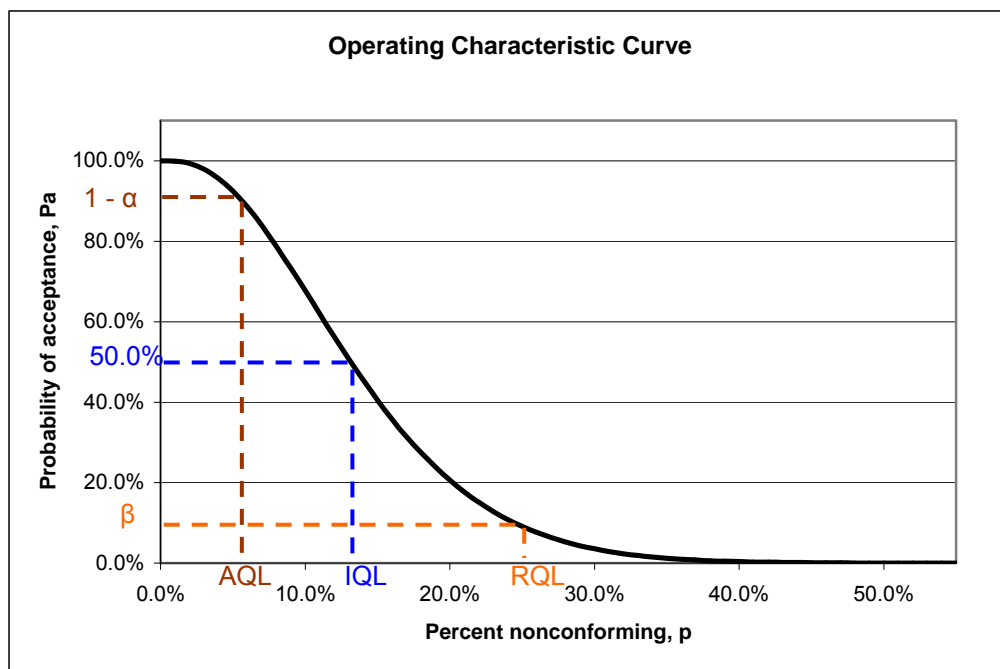Figure 5 illustrates these points.

Figure 5 Specific Points on the OC Curve

A Stream of Lots and the Binomial Distribution
We described the OC curve in terms of a process that produces a series of lots. This leads us to recognize that the underlying distribution is the binomial. In the binomial distribution, there are two possible outcomes. The items in the sample are either conforming or nonconforming. In addition, the probably of selecting a nonconforming item doesn't change as a result of the sample. Since we are sampling from a process, the potentially infinite number of items is not impacted by taking the sample.

When the producer presents lots for acceptance, they often come from a process that is operating at some quality level, *i.e.,* the process produces a certain percentage of nonconforming items. The probability of obtaining a specified number of nonconforming items, *Pr(x)*, from a sample of *n* items with percent nonconforming, denoted *p*, is given by the binomial distribution.

$$\Pr(x) = \binom{n}{x} p^x (1-p)^{n-x}, \; x = 0, 1, \ldots, n$$

In a single sample plan we accept the lot if the number of nonconforming items is c or less. This means we are interesting in the probability of 0, 1, …, c items. We write this as

$$\Pr(x \leq c) = \sum_{i=0}^{c} \binom{n}{i} p^x (1-p)^{i-x}$$

The probability of accepting the lot is the probability that there are c or fewer nonconforming items in the sample. This is the equation above, and is what we plot as the OC curve.

The Isolated Lot And The Hypergeometric Distribution
The binomial distribution applies when we consider lots coming from an ongoing production process. Sometimes we consider isolated lots, or we are interested in a specific lot. In these cases, we need to realize that taking the sample, because we sample without replacement, changes the probability of the next item in the sample. In these cases, we need the hypergeometric distribution.

$$\Pr(x) = \frac{\binom{d}{x}\binom{N-d}{n-x}}{\binom{N}{n}}$$

Here, N is the lot size, n is the sample size, and d is the number of nonconforming items in the lot.

If we are interested in determining the probability of c or fewer nonconforming items in the sample then we write:

$$\Pr(x \leq c) = \sum_{i=0}^{c} \frac{\binom{d}{i}\binom{N-d}{n-i}}{\binom{N}{n}}$$

We can use this equation to draw the OC curve for the isolated lot.

Often, the β risk is applied to each lot, instead of the stream of lots. In these cases, the quality level corresponding to a probability of acceptance equal to β is called the Lot Tolerance Percent Defective (LTPD).

Single and Double Sample Plans
The material above discusses sampling plans in which we draw one sample from the lot. This is called a single sample plan. We describe the plan by a set of parameters:
  n is the sample size,

c is the maximum number of nonconforming items allowed for acceptance, and
r is the minimum number of nonconforming items allowed for rejection.

In a single sample plan r and c differ by 1.

In contrast, there are double sampling plans in which we take the first sample and make one of three decisions: accept, reject, or take a second sample. If we take the second sample, we then make an accept/reject decision.

As described above the set of parameters used to describe a double sample plan are:
n_i is the ith sample size,
c_i is the maximum number of nonconforming items allowed for acceptance on the ith sample, and
r_i is the minimum number of nonconforming items allowed for rejection on the ith sample.

For example, a single sample plan may be:

$$n = 20, c = 2, r = 3.$$

A double sample plan may be:

$$n_1 = 20, c_1 = 1, r_1 = 4$$
$$n_2 = 20, c_2 = 4, r_2 = 5$$

In this example, if we had 2 nonconforming items on the first sample, we would draw the second sample. In total, we would have sampled 40 items.

We can calculate the probability of acceptance, the information we need to define the OC curve, by the following equation.

$$\Pr(x_i \leq c_i) + \sum_{i=c_i+1}^{r_i-1} \Pr(x_1 = i) \times \Pr(x_2 \leq c_2 - i)$$

The probabilities are, as described above, calculated using either the binomial or hypergeometric distributions.

The c=0 Sampling Plans
Many practitioners are concerned that traditional lot acceptance sampling plans allow nonconforming items in the sample. For example, the single sample plan *n=20, c=2, r=3* allows as many as two nonconforming items in the sample.

One solution is the use of plans that don't allow any nonconforming items. One example of plan is *n=20, c=0, r=1*. The consumer would reject the lot if any nonconforming items appeared in the sample.

Sampling plans with c=0 don't have the same kind of OC curve discussed above. Instead of the classic "S" shape, that starts to approximate the ideal curve, c=0 OC curves drop off sharply without the bend. Figure 6 shows the OC curves for these two plans.

Notice how quickly the c=0 plan drops off. The figure also has dashed horizontal lines at 5% and 95% probability of acceptance. The AQL and RQL for these plans are listed below.

	c=0	c=2
AQL	0.26%	4.21%
RQL	13.9%	28.3%

It is easy to see that c=0 plan will accept many fewer lots than the corresponding c=2 plan. If your process cannot tolerate even a few nonconforming units, c=0 plans may be a good approach. However, recognize that lot rejection incurs a transaction cost, that may be high. The selection is a c=0 plan is certainly an economic decision.
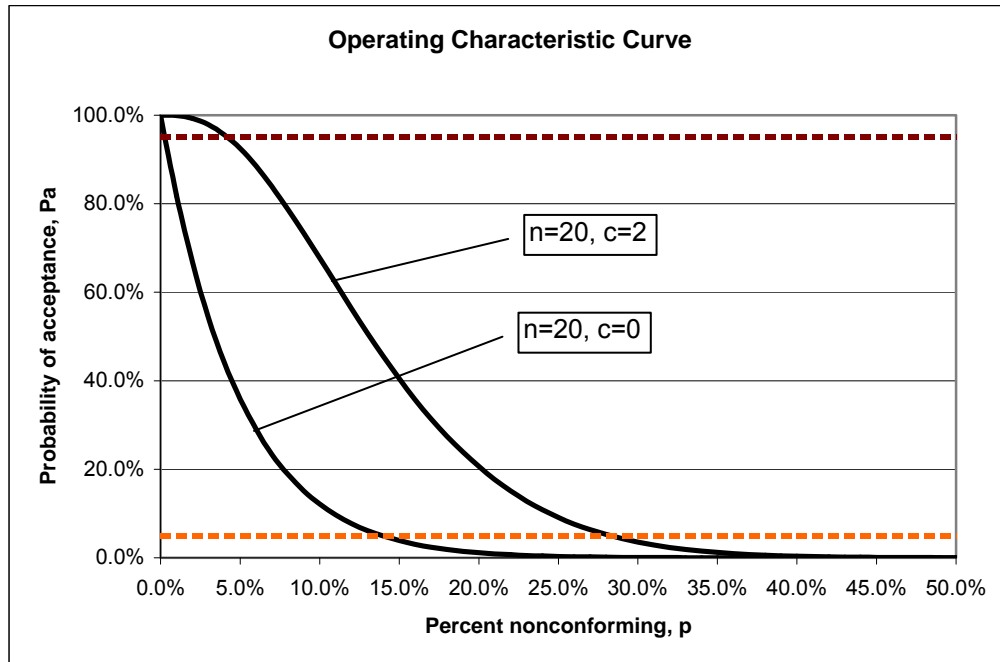
Operating Characteristic Curve

Figure 6 OC Curve comparison Showing c=0 Effect